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When was Ptolemy's Star Catalogue in 'Almagest' Compiled in Reality? Statistical Analysis

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Abstract. This work is devoted to describing new statistical and geometrical procedures for dating ancient star catalogues considering numerical data contained in these catalogues and the known real configurations of stars on the celestial sphere. The method was tested on several star catalogues whose dates are well known (Tycho Brahe, etc.) and on several artificially generated star catalogues. Then the same method was applied to the Almagest. The results obtained do not confirm the traditional dating of the Almagest (2nd century AD or 2nd century BC) but shift its dating to the Arabian epoch: 600–1300 AD.

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0. Preliminary Remarks on the Mathematical Aspects of the Problem

This paper involves at least three aspects: historical (Section 1), astronomical (Sections 2 and 3) and mathematical (Sections 4–8).

Its main problem is: 'Is it possible to date an ancient star catalogue by mathematical tools?' Our answer is: 'Yes'.

The initial data are star coordinates contained in the star catalogue. In order to determine the date of its compilation, we need to know the correct star coordinates for any point of time in the past. For this purpose, we use the well-known equations describing the trajectories of stars starting from their present positions. Every star from the catalogue has two coordinates – longitude and latitude. We consider in our work only latitudes. The reason for this is that it is possible to obtain all the necessary information for dating using latitudes only. Longitudes are not reliable for this purpose (see details in Section 1). They lead to additional errors, and eliminating them provides additional accuracy.

Mathematical problems arise in connection with the comparison of the real positions of the stars in the past with the coordinates (latitudes) contained in the catalogue. They arise because of the low accuracy of the catalogue and the different types of estimates presented in it. In fact, the catalogue contains more than 1000 stars but they cannot be considered as homogeneous random samples. This means that different groups of stars can have different types of errors.

So the main part of our analysis consists of the classification of the various errors (see Section 5). The first problem in classification is the following problem: 'Who is who' in the star catalogue? (see Section 4). The fact of the matter is that some of the stars cannot be identified on a one-to-one basis with definite 'modern' stars. So we deleted such stars from the catalogue, as any dating procedure cannot be based on them.

The second problem is implied by so-called 'spikes'; often due to copying errors or neglect of refraction effects, which represent 'large deviations' of the catalogue's coordinates from corresponding real values (see Section 5.3). Such large deviations must also be deleted, as they do not contain any useful information for dating.

The third problem is that the coordinates of some large groups of stars can contain 'systematic errors' due to errors in the measuring (or calculating) of the position of the ecliptic (Section 5.2). These errors lead to some definite shift of latitudes (see formula (2)). These shifts can be determined by solving a special spherical regression problem (see Section 6). A solution of this problem gives us the possibility to compensate for such systematic errors. After this, we can estimate the residual error by considering latitudes of the stars from the group as a random sample. Of course, before doing this, we need to define 'homogeneous' groups of stars (see Sections 6.3 and 6.4). These residual errors can be treated as errors of measurements.

After the classification of errors, deleting some 'suspicious' stars, compensating for the systematic errors, and defining the errors of measurements we have a good idea of the 'statistical accuracy' of the catalogue.

But for dating we need to consider stars with high proper velocity and which also have small individual (not statistical!) errors. For this, we suggest taking the so-called 'named stars' (Arcturus, Procyon, etc.). If there are really small individual errors in their data, then we can find such a moment of time when all these stars have small individual deviations (less than $10'$ – the value of the accuracy of the catalogue). It turned out that such times do exist and they belong to an interval from AD 600 till AD 1300. This is an interval of possible datings (see Section 7.1).

The above result relies on statistical arguments which therefore have a small non-zero probability of being wrong. So the question arises: "Is it possible to find a rotation of the celestial sphere (for given time t not belonging to the above interval) leading to a difference between all real latitudes of the chosen named stars and corresponding latitudes from the catalogue which is less than 10?' Geometrical arguments say 'No'. So, the interval from AD 600 to AD 1300 just mentioned cannot be enlarged (see Section 7.2). We studied the stability of the proposed method in respect to different perturbations of the initial suppositions and data (content of the group of named stars, accuracy of the measurements etc.). It turned out to be stable (Section 8).

Finally, all the procedures were tested on well-dated star catalogues (Section 9). In all cases, the calculated interval contained the real compilation date.

1. History of the Problem and Subject of the Work

Interest in the dating of the Almagest (compiled by Ptolemy [21]) is not new. One can find a review of the relevant problems in *The Crime of Claudius Ptolemy* by the well-known astronomer R. R. Newton [2] as well as in a fundamental investigation by C. H. F. Peters and E. B. Knobel [1]. Investigations by N. A. Morozov in 1928 [6] contained well-argumented objections to the traditional dating of the 2nd century AD or the 2nd century BC as a real dating of the Almagest. Many interesting and critical pieces of material are contained in the book of R. R. Newton, mention above. Newton formulated many conjectures to the effect that the main part of the astronomical data in the Almagest have been falsified but he adopted a traditional version of dating.

New methods to attack dating and chronology problems were introduced and applied in a series of papers by one of the authors (A.T.F.) [7–15, 17, 18]. When applied to the particular case of the dating of the Almagest, the methods gave a result that is quite different from the traditional compilation dates. Other statistical methods by other statisticians give compatible if less precise results [24].

The recent paper by Yu. N. Efremov and E. D. Pavlovskaya [16] is devoted to an attempt to confirm a traditional dating of the Almagest star catalogue on the basis of the proper motions of some stars. But it is not correct in many places and this attempt cannot be considered as a serious one.

As the proper motions of modern stars are known today with great accuracy, it is possible to calculate their positions in the past and compare them with corresponding ones taken from an ancient star catalogue. In principle, this then permits the determination of the date of its compilation. However a 'straightforward' approach is not successful here because of the low accuracy of ancient catalogues and slow speed of the proper motion of most stars. This is the reason that led to the developing of new methods for dating such catalogues.

Below, we propose geometrical and statistical procedures which have been tested on several star catalogues with well-known compilation dates and on some artificially compiled star catalogues. For the latter case, the 'date of observation' was of course known to the compiler but not to a 'researcher'. These procedures appear to be rather accurate: all dating intervals calculated with their help covered real (known) dates. The same method was then applied to the Almagest star catalogue. The results obtained do not confirm the traditional dating of this catalogue.

Our work (carried out in 1985–1988)) is based on careful analysis of all geometrical, statistical and numerical aspects of the problem. We do not touch here on any historical problems. The work is purely mathematical. The method is based on an analysis of numerical data contained in the underlying star catalogue, namely on the analysis of star coordinates. Hence, *our paper deals with the star catalogue only* (not the Almagest as a whole) and all our conclusions only refer to the star catalogue.

R. R. Newton [2] showed that longitudes are not reliable data in the Almagest and so the main question we tried to answer is: 'Is it possible to date star catalogues using only latitude values?' Our answer is 'Yes'. We insist on separating this answer from different chronological problems which do not have precise mathematical descriptions.

2. Some Notions from Astronomy

Let us introduce some stardard notions (see [1, 2] and Figure 1). Suppose that stars belong to a celestial sphere with its center in the 'eye of an observer'. To fix the positions of the stars, we need a spherical coordinate system. Two such systems were usually used in the Middle Ages: the equatorial system and the ecliptical one. The equator of the celestial sphere is the circle resulting from an intersection of the sphere with the plane of the Earth's equator. Parallels and meridians can then be introduced onto the sphere. Equatorial latitude δ is measured in arc degrees $(-90° \leqslant \delta < 90°)$ and is called the *declination* of the star. Equatorial longitude α is measured in hours $(0 \leqslant \alpha \leqslant 24\,\text{h})$ and is called the *ascent* of the star. Of course, a starting point for the longitudes must be determined – see below.

The intersection of the celestial sphere with the plane of the Earth's orbit is called the ecliptic. The constellations of the Zodiac are placed along the ecliptic. We can now define new (ecliptical) latitudes and longitudes. Ecliptical latitude b is measured in arc degrees $(-90° \leqslant b \leqslant 90°)$, ecliptical longitude l – also in arc degrees $(0° \leqslant l \leqslant 360°)$. The intersection of the equatorial plane with the ecliptical one is the axis of the equinox, see OC in Figure 1. This axis intersects the celestial sphere at two points – the spring equinox and the fall equinox. The point of the spring equinox is taken as 'zero meridian' for both equatorial and ecliptical longitudes.

These two coordinate systems are not fixed. They evolve in time for the following reasons:

(a) The axis of the Earth's rotation (see ON in Figure 1) moves approximately along a cone surface with a vertex angle equal to about $23° 27'$ (in AD 1900). In Figure 1, this is the angle between ON and OP. This motion is called the precession and its velocity is about $50''$ per year. Consequently,

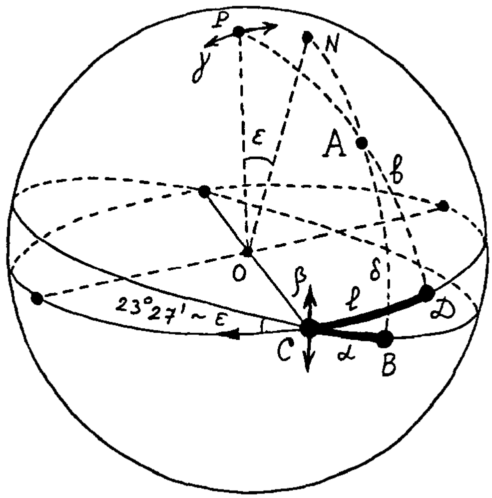

Fig. 1.

the equatorial coordinate system and the axis of the equinox have a precession which induces a precession of the ecliptical longitudinal (see point C in Figure 1). If we fix a point on the celestial sphere, then its ecliptical longitude $l(t)$ varies with t approximately uniformly.

(b) The Earth's axis has small oscillations (so-called nutations), their maximum amplitude being less than 17″

(c) The third important perturbation is the oscillation of the ecliptic. This oscillation is introduced by oscillations of the plane of the Earth's orbit in time. Let $\epsilon(t)$ denote an angle between planes of ecliptic and equator, see Figure 1. The function $\epsilon(t)$ describes the ecliptic oscillations dependent on time t.

Here we take into consideration the precession and ecliptic oscillation effects but not nutations (because of their negligible small values).

An astronomical and mathematical theory permitting the calculation of star positions in the past and future, taking into account the effects mentioned, was suggested by Newcomb. This theory is well known and generally accepted. It is the basis of all modern calculations concerning star dynamics. We used Newcomb's theory in the form of H. Kinoshita [3] for computer calculations of the real positions of stars in the past.

Proper motions of stars have been taken into account too. All data about the directions and velocities of stars were taken from the catalogues[4] and [19]. The most visible stars (mentioned in the Almagest) move very slowly, but stars exist (bright ones) whose positions on the celestial sphere have changed by several degrees for over a period of more than 2000 years.

We measure time t using centuries as units. So the value $t=0$ corresponds to AD 1900 as the coordinates of 'modern' stars refer mainly to this year. The value

$t = 1$ corresponds to AD 1800, the value $t = 3.75$ corresponds to AD 1525, and so on. The parameter t runs over some a-priori fixed time interval. For the Almagest, we chose the interval from 600 BC till AD 1900, i.e. $0 \leqslant t \leqslant 25$.

3. Some Characteristics of Ancient Star Catalogues

We studied the star catalogues of Ptolemy, Tycho Brahe, Ulugbek, Al-Sufi, and Hevelius. They were all compiled without the aid of a telescope. Each catalogue contains about 1000 stars. Modern catalogues (compiled with the help of a telescope) contain many more stars and use equatorial coordinates because they can be measured more simply and accurately than ecliptical ones. But Medieval and ancient catalogues use ecliptical coordinates. Ancient astronomers had no idea about small ecliptic oscillations and, hence, supposed the ecliptical coordinates to be 'eternal' ones. In other words, they believed ecliptical latitudes had not been changing over time while longitudes had been changing with constant precession velocity. Equatorial coordinates vary in a much more complicated way (even for fixed stars). After the discovery of the effect of the ecliptic oscillation, all 'advantages' of ecliptical coordinates disappeared.

Some stars from ancient catalogues have proper names – the named stars. usually, these are very bright stars. There are some stars with considerable proper motion among them, e.g. Arcturus, Procyon, Sirius. It is natural to suggest that proper names were given to important famous stars and their coordinates were measured with special care. In general, the list of named stars can depend on the underlying catalogue.

The accuracy of a catalogue is a very important thing for its dating purpose. It is natural to suggest that the claimed accuracy (i.e. the scale) corresponds to the real one. It is useful to mention that the scale of the Almagest star catalogue is 10′, the scale of T. Brahe's catalogue is 1′ and Hevelius' catalogue is 1″, see [5]. But numerous investigations (see [2]) force us to conclude that the real accuracy of ancient catalogues can be worse than the claimed one. For example, R. Newton [2] proves that the mean-square latitude error in the Almagest is 20′ not 10′ and that the error in the arc deviation is equal to 1° 12′. The last term contains some systematic error. After compensating for that the arc error decreases to 23′. The accuracy of T. Brahe's catalogue is considered to be 2′–3′ (but not 1′) by modern specialists. This fact was also confirmed by our calculations. It is reasonable to suppose that the accuracy of Hevelius' catalogue is close to that of T. Brahe since the two observers used practically the same instruments. That is the accuracy of Hevelius' catalogue can not be 1″ but is about 2′. This hypothesis is confirmed by our calculations.

We shall not discuss all the numerous possible reasons for the appearance of errors in ancient catalogues, but refer the reader to R. Newton's book [2]. We do not really need to consider them. All we need to know are the consequence of these reasons. Here we list some important facts.

(a) An analysis of methods used for measuring coordinates shows that, for ancient catalogues, possible latitude deviations must be statistically less than longitude deviations. In other words, the latitudes presented in ancient catalogues are the most accurate coordinates.
(b) Longitude deviations can include some additional terms resulting from possible recalculations of the catalogue in order to take the precession effect into account [21].
(c) Medieval and ancient star catalogue compilers were not aware of the refraction effects nor the summing of errors in the observation processes. Such errors actually do occur in their catalogues.
(d) Some errors in catalogues was introduced by copyists. In original manuscripts of the Almagest, letters were used to denote figures and this caused difficulties in the interpretation of numerical data.

If we consider errors in coordinates to have a random nature, then (within the limits of the real accuracy of the catalogue) we can treat them as random variables chosen from some homogeneous sample (e.g. normal). 'Large deviations' or 'spikes' can be attributed to the causes listed above (see (c) and (d)). The hypothesis of randomness is unnatural for 'spikes' leading to the necessity to examine all suspicious cases individually. Final conclusions cannot be based on considerations of such 'suspicious' stars, so they must be deleted. Several such cases are discussed in [1, 4] and we have considered similar cases carefully.

4. Preliminary Analysis of the Almagest

We base our work on the 'canonical' version of the Almagest star catalogue as is presented in the fundamental work of Peters and Knobel [1]. At the starting point of our investigation, we doubted neither star coordinates nor the traditional assumption that their ecliptical coordinates correspond to the year AD 60.

Identification of dim stars from the Almagest catalogue with modern ones is a complicated problem which cannot be solved definitely in all cases. Stars in the Almagest are listed by means of their coordinates and some verbal description. Identifications with modern stars were made by different authors, see [1]. Sometimes these identifications are not 'one-to-one'. In order to meet our demand for reliable data, we have to solve the identification problem anew. For this purpose, we chose from the modern star catalogue a set of about 30 named stars and 50 'fast' stars (within a velocity of proper motion $v > 0.5''/1$ year). For solving the 'Who is who'' problem Newcomb's theory was used. Namely, we calculated (using a computer) the ecliptical coordinates of all mentioned stars varying t from 0 till 25 (i.e. from 600 BC till AD 1900). Then these coordinates were compared with those given in the Almagest.

The most traditional identifications were confirmed. But we discovered several modern stars (e.g. O^2 Eridanus) which can be identified with different Almagest stars in different epochs. In other words, the identification of such stars (and, consequently, the answer to 'who is who?') is a function of time t. For O^2 Eridanus we found the following different possibilities: 778, 779, 780 (in Baily's numeration [1]). Note that one can find some doubts in [1] about the identification of O^2 Eridanus. These facts completely undermine the paper by Efremov and Pavlovskaya [6] mentioned above, since the proper motion of O^2 Eridanus is the basic argument in it for discovering the dating of the Almagest. In reality, Efremov and Pavlovskaya first suppose that the Almagest was indeed compiled in the 2nd century BC and then 'prove' that it is true. In our opinion, such 'doubtful' stars as O^2 Eridanus must be excluded from consideration; changes in the identification of such stars imply changes in the dating result.

After having completed the identification procedure, we obtained a list T of all stars having reliable one-to-one identification with the stars of the Almagest. This list contains the following information: (1) Baily's number i; (2) the ascent α_i and declination δ_i of the ith star from the modern catalogue at time $t = 0$; (3) the velocity components of the proper motion of the ith star; (4) the ecliptical longitude l_i and latitude b_i for the ith star taken from the Almagest.

Let $L_i(t)$ and $B_i(t)$ denote the ecliptical coordinates of the ith star in time t as calculated according to Newcombe's method. The problem of dating is then reduced to finding t_0 such that the set of coordinates

$$V(t_0) = \{L_i(t_0),\ B_i(t_0)\}$$

is the closest in a sense to the set of the Almagest's coordinates $V_A = \{l_i, b_i\}$. The primitivity of this idea can be only compared with the difficulty of its solving. Overcoming the obstacles which arise is the subject of the present work.

Usually such a problem can be solved by choosing a natural distance between the sets $V(t)$ and V_A. Then one can determine a moment t_0 when this distance takes a minimal value. In our case, it appears that a possible error in the calculation of t_0 is too large. For example, let $a_i(t)$ be the arc distance between the ith star with coordinates $(L_i(t),\ B_i(t))$ and (l_i, b_i) and let $t_i^* = \mathrm{argmin}(a_i(t))$. It is easy to see that if the coordinates of this star in Almagest have an error Δ and if v_i is its velocity, that the error in the determination t_i^* is about Δ/v_i. Consequently, we can state only that the date t_0 belongs to the interval $(t_i^*{}_{-\Delta/vi,\,t_i^*} + \Delta/v_i)$. For example, in the case of the Almagest (using the most optimistic estimates), we have $\Delta \approx 14'$ and $v = 1.5''/\text{yr}$. Here $14' \approx ((10')^2 - (10')^2)^{1/2}$, $10'$ is the claimed exactness of the Almagest star catalogue, and $1.5''/\text{yr}$ is the velocity of a very fast star (Arcturus). Thus, we see that the time interval of possible solutions for this case is equal to about 1200 years – this result also contradicts those obtained in [16].

Our numerical investigation confirmed the lack of accuracy of 'point minimum' methods. It appears that by only slight variations of the initial data (e.g. by changing of the set of underlying stars), we can shift the point of minimum from $t = 0$

to $t = 25$. Moreover, it turned out that the final result depends on the kind of distance used. This means that such results are extremely subjective.

A set of named stars in the Almagest consists of 12 stars designated 'vocatur' in the text. They are, with their modern names and Baily numbers in parentheses, Arcturus (α Boo, 110), Sirius (α C Ma, 818), Aquila (α Aql, 288), Previndemiatrix (ϵ Vir, 509), Antares (α Sco, 553), Aselli (γ Cnc, 452), Procyon (α C Mi, 848), Regulus (α Leo, 469), Spica (α Vir, 510), Lyra (α Lyr, 149), Capella (α Aur, 222), Canopus (α Car, 892).

Table I shows the deviations in latitudes, $|B_i(t) - b_i|$, for all these stars (in minutes) for several values of t.

The values $t = 18$ and $t = 21$ correspond almost exactly to the traditional dates for the lives of Ptolemy and Hipparchus. (Recall that some experts attribute the Almagest to Hipparchus.) Table I confirms that it makes no sense to date the catalogue using the exact minimum of the usual distance between stars or between star configurations.The 'minimal' point is very sensitive to small variations in the initial date or set of stars.

Table I also shows that for seven (of the 12) stars, the latitude deviations $|B_i(t) - b_i|$ are more than 10′ for all t from a-priori time interval. For Spica, this deviation is less than 10′ for all t. If we turn to the other four stars, we see that not more than two stars have 10′ deviations simultaneously (for all t!). This fact is extremely surprising, since it is valid for the bright, named (i.e. famous) stars, the very ones whose coordinates must have been measured most carefully. It implies that the Almagest star catalogue must contain some systematic error.

Table I. Deviations in latitudes for the 12 vocatur stars

	t					
No.	1	5	10	15	18	21
110	37.6	21.2	0.9	19.0	31.4	43.3
818	23.6	18.3	11.7	5.1	1.2	2.6
288	8.6	9.4	10.5	11.8	12.6	13.4
509	13.0	14.3	15.8	17.1	17.8	18.4
533	32.6	29.5	25.5	21.6	19.3	17.0
452	30.5	28.5	25.9	23.2	21.5	19.8
848	11.2	16.0	21.9	27.6	31.1	34.4
469	17.5	16.6	15.4	14.0	13.0	12.1
510	2.4	0.7	1.3	3.1	4.2	5.2
149	15.4	14.2	12.5	10.8	9.8	8.7
222	21.9	21.7	21.3	21.0	20.8	20.6
892	51.0	54.2	58.2	62.3	64.8	67.3

5. Errors in the Almagest Star Catalogue

5.1 TYPES OF ERRORS OCCURRING IN THE CATALOGUES

We have already noticed that it is necessary to analyse all possible types of errors in the catalogues. We divide these errors into three types: systematic, random, and spikes.

Systematic errors appear as a result of global measurements or recalculations which imply a global rigid rotation of some sets of stars on the celestial sphere. Such systematic errors do occur in the Almagest (see below).

Random errors result from mistakes in individual measurements. Errors of this kind cause a random movement of each star on the celestial sphere. It is reasonable to assume that these random errors have a distribution with zero mean value. Such errors do not usually exceed the size of the scale unit (of the instrument).

Spikes are caused by circumstances beyond the control of the observer and unknown to him – e.g. the errors of later copyists, refraction, etc. These errors usually change coordinates much more than one unit of the scale and occur rarely.

5.2. SYSTEMATIC ERRORS

Systematic errors are most frequently caused by a recalculation of equatorial coordinates into ecliptical ones. Such a recalculation was inevitable, since all astronomical instruments were installed on the Earth (at least, we suppose this) and initially were tied with the equatorial system.The transition to the ecliptical system can be realized with the aid of mathematical formulas, special globes, or astronomical instruments. Thus, the term 'recalculation' is interpreted here very broadly. In any case, in order to compile a catalogue in terms of ecliptical coordinates, the astronomer must know the position of the ecliptic and the position of the equinox axis OC in the epoch to which the catalogue is reduced (Figure 1). This position is generally known only with some error τ_1. The error in calculation of the point C (along the ecliptic) implies the systematic rigid translation of the longitudes of all stars which have been measured with the help of that calculation. Next, the astronomer can make a mistake τ_2 in the definition of the longitudes of all stars simultaneously. These two errors are to be summed to obtain the systematic error in longitude $\tau = \tau_1 + \tau_2$.

The next possible systematic error is caused by a translation of the equinox point C along the meridian. In other words, it is an error in the latitude of C. This error is denoted by β in Figure 2. Instead of β, we can introduce a parameter φ which is an angle between the real axis of the equinox and the intersection line of the equatorial plane with the 'catalogue's ecliptical plane, see Figure 2.

The two errors, β and τ (or φ and τ), totally describe all possible shifts of the point C on the celestial sphere: any error is a combination of β and τ.

The third error (which we denote γ) can appear due to an erroneous calculation of the angle ϵ between the equator and the ecliptic (Figure 1). In reality, the

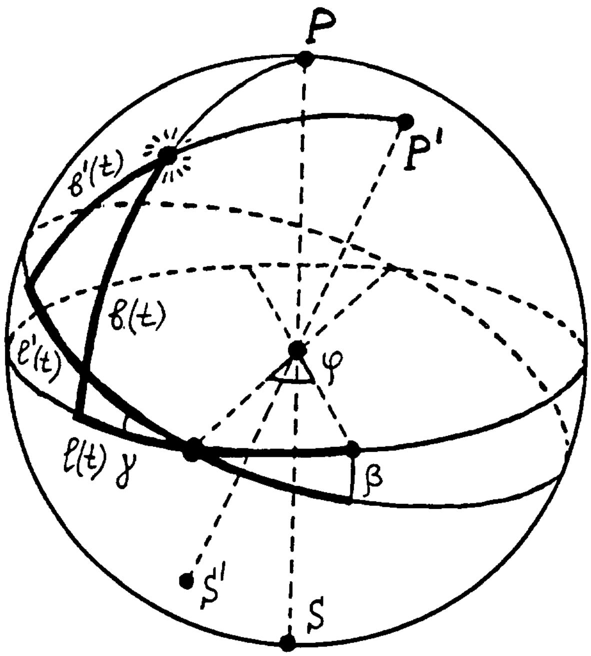

Fig. 2.

parameters γ and β (or γ and φ) completely define a position P′ of the pole of the ecliptic on the celestial sphere, see Figure 2.

It is clear that any rotation of the celestial sphere can be decomposed into the composition of three orthogonal rotations defined by the parameters τ, β and γ. Thus, they reflect all possible systematic errors (if any). The possibility of systematic errors in the Almagest has been discussed by many authors, see [1, 2, 6]. Let us summarize the results of these discussions.

An error τ can be induced by the observer's attempt to base a star catalogue at some date other than the actual date of observation. The catalogue of T. Brahe, for example, was based at AD 1600 (while it was observed about three decades earlier). It is easy to hide the real date of observation by adding some value to the longitudes of all stars [2, 6, 21, 22]. Sometimes the error τ is a consequence of a change in the position of 'zero meridian': ancient astronomers did count longitudes starting from different initial points on the ecliptic – Copernicus, for example.

What about errors β and γ? The equatorial latitudes can be determined from observations in a very simple way (see [2]) so that we can assume that the error β at the time of actual observation must be practically zero. The error γ has a quite different character. An accurate determination of the ecliptic position requires complicated calculations or/and nontrivial observations and measurements so that the values of γ must be considerably greater than those of β. Of course, $\sin\beta = \sin\gamma = \sin\varphi$. References [1] and [6] discuss possible values of γ. They estimate it to be about 20′–30′. Our calculations confirmed this fact. If we assume that β is about 5′, then φ is about 15°.

5.3. RANDOM ERRORS AND SPIKES

Let us consider a star from our list T with Baily's number i (l_i and b_i being its ecliptical coordinates). We denote by $L_i(t, \tau, \beta, \gamma)$ and B_i (t, τ, β, γ) the ecliptical longitude and latitude of this star at time t given that the systematic errors are equal to τ, β and γ. Similarly, we introduce notations L_i $(t, \tau, \varphi, \gamma)$ and $B_i(t, \tau, \varphi, \gamma)$. Here and below, we can always substitute β for φ if this is desirable and we shall do that without any explanation. Now it is possible to compare coordinates l_i, b_i from the Almagest with those above. Let us consider the following latitude and longitude deviations:

$$\Delta_b(i, t, \beta, \gamma) = B_i(t, \beta, \gamma) - b_i,$$

$$\Delta_l(i, t, \tau, \beta, \gamma) = L_i(t, \tau, \beta, \gamma) - l_i.$$

Here we used the obvious fact that the latitude (and, hence, the latitude deviation) does not depend on τ, i.e.

$$B_i(t, \tau, \beta, \gamma) = B_i(t, \beta, \gamma).$$

This is one reason why latitudes are more informative than longitudes. We shall mainly use latitudes in our calculations (which do not depend on the systematic error τ) and consider longitudes only as auxiliary data.

If measurements for the ith star do not contain some unforseen errors (copyist's mistake, refraction, etc.) then deviations Δ_b and Δ_l must be within the scale of the underlying catalogue. The real accuracy of a catalogue can be unknown. Moreover, it may be that the author chose his 'record' accuracy as the catalogue scale unit; that is the accuracy of the observations of the most famous stars. In order to find and eliminate 'spikes', we can use the following method (given that the values β and γ are fixed):

(1) Find a mean-square deviation

$$\delta = \left[\sum_{i=1}^{N} \Delta_b^2(i, t, \beta, \gamma) \right]^{1/2},$$

where N is the number of stars in the list T. In fact, the value δ does not really depend on t since most of the stars have a negligible proper motion. Thus, we can take the resulting value δ or even $\delta/2$ as a 'record' accuracy Δ of the given catalogue. The 'real' accuracy is equal to some value between 2δ and 3δ. We note, too, that about 40% of the stars are within the record accuracy interval.

(2) Stars whose coordinates are not within the real accuracy must be excluded from further analysis. Either these stars are 'spikes' or they acquired large errors in the measurement of their coordinates.

Since there are few such 'spikes', they do not affect δ. We excluded from the list T also all stars whose coordinates were considered by previous researchers to be doubtful, see Peters and Knobel [1]. So the stars whose coordinates were deformed by refraction (e.g. Canopus) were detected from T.

6. Statistical Analysis of the Almagest Star Catalogue

6.1. PRELIMINARY REMARKS

The star catalogue in the Almagest contains 1025 stars. Their coordinates (ecliptical longitudes and latitudes) are given in the catalogue with a 'claimed accuracy' of 10′, i.e. the author believed that he really reached an exactness of 10′. All stars are collected in constellations which are arranged in a natural order from north to south.We have studied a 'canonical' version of the catalogue from a fundamental work [1] which contains, in particular, results of the identification of the stars from the Almagest with 'modern' stars. As we mentioned above, some 'fast' stars had to be deleted from the catalogue because of their uncertain identification. One can find in [1] real errors in coordinates of stars from the Almagest star catalogue. These errors have been obtained by Peters, given that the dating of the Almagest is about AD 100. Although these calculations do not completely fit our situation, they can be used for deleting some 'large deviations' (more than 1°). We pointed out that such 'doubtful' stars are not informative. As a result, we obtained a 'clean catalogue' which contains 864 stars. This served as the subject of our statistical investigations.

It is interesting to note that two stars (Canopus and Previndemiatrix) which were removed from the catalogue, turned to be a spikes, see [20] for details.

Let again l_i and b_i be the ecliptical longitude and latitude of the ith star from the clean catalogue. Let $L_i(t)$ and $B_i(t)$ be real corresponding values for time t. A detailed and careful statistical analysis shows (see [2]) that longitudes in the Almagest cannot be considered reliable numerical data. R. Newton showed in [2] that these data were the result of some complicated recalculations of the initial ones. But all specialists agree that latitudes are the initial observed data. We based our investigation on latitudes only. It turned out that analysis of latitudes only gives us the possibility of separating all stars into groups having 'well-measured' coordinates and groups having 'badly measured' ones. *We demonstrate* in this paper *that star catalogues* (not only the Almagest but many others!) *can be dated with help of latitude data only.*

Recall that the initial mean-square errors of star latitudes in the Almagest,

$$\sigma = \left[\left(\sum_{i=1}^{N} (b_i - B_i(t))^2\right)\Big/ N\right]^{1/2},$$

is equal to approximately 20′. This accuracy does not really depend on time t $(0 \leq t \leq 25)$.

6.2. A CLASSIFICATION OF LATITUDE ERRORS

Let t^* be the real (but unknown to us) year of the observation of the stars. We started with a decomposition of the real latitude deviation$\Delta b_i(t^*) = b_i - B_i(t^*)$ in two components:

$$\Delta b_i(t^*) = \xi_i + r_i(t^*). \tag{1}$$

Let us call the value ξ_i the error of observation. It can be inspired by many various causes and there is no reason to discuss them here. it is natural to suggest that ξ_i is a Gaussian random variable with zero mean value, $E\xi_i = 0$, and with finite variation $d = E\xi_i^2 = 0$. We can call the component $r_i(t^*)$ an error due to a wrong determination of the ecliptic pole. The position of the ecliptic was known to ancient astronomers with some error which can be characterized by the two parameters γ and φ, see Figure 2 and Section 5. From the definitions it is easy to obtain that

$$r_i(t^*) = \gamma_i \sin(L_i(t^*) + \varphi_i) + \delta_i, \tag{2}$$

where $|\delta_i| < 1''$ if $|B_i(t^*)| < 80°$. Consequently, the value δ_i can be neglected in our calculations.

The idea of the proposed method is the determination of γ_i and φ_i by mathematical statistics and to compensate for these errors in order to deal with the real observation error only. Such an approach leads us to a dating method. The realization of the method is based on the fact that the parameters γ_i and φ_i have a 'group-like nature', i.e. they are the same for certain groups of stars (e.g. for constellations). This is really true in many cases because γ_i and φ_i do not depend on individual measurements but on preliminary determination of the ecliptic position for the groups mentioned.

We assume that each constellation G in the ancient catalogue has an individual group error (i.e. this error is common for all stars from the constellation) in the determination of the position of the ecliptic pole. Let us parameterise it by values γ_G and φ_G. That is, for each star $i \in G$, we assume that equalities $\gamma_i = \gamma_G$ and $\varphi_i = \varphi_G$ are true. Our aim is to estimate γ_G andφ_G for each group G from the catalogue. Note that the Almagest star catalogue contains 48 constellations.

6.3. ANALYSIS OF ERRORS. SEVEN REGIONS IN THE ALMAGEST STAR ATLAS

Let us suppose that t is the year of observation. Determine the value

$$\Delta b_i(t, \gamma, \varphi) = b_i - B_i(t) - \gamma_G \sin(L_i(t) + \varphi_G) \tag{3}$$

for the ith star and consider a constellation G containing N stars. Then we calculate values for $\hat{\gamma}_G$ and $\hat{\varphi}_G$ from the condition of minimization of the function

$$\sigma_G^2(t, \gamma, \varphi) = \left[\sum_{i=1}^{N_G} \Delta b_i^2(t, \gamma, \varphi)\right] \Big/ N_G \rightarrow \min, \tag{4}$$

varying γ and φ. This problem can be easily solved analytically.

Let us call the value

$$\delta_G^{\min}(t) = \delta_G(t, \hat{\gamma}_G, \hat{\varphi}_G)$$

a minimal mean-square error in the constellation G. We additionally calculate the percentage $p_G^{\min}(t)$ of stars from G which meet the inequality $|\Delta b_i(t, \hat{\gamma}_G, \hat{\varphi}_G)| < 10'$, i.e.

$$p_G^{\min}(t) = [\#\{i : |\Delta b_i(t, \hat{\gamma}_G, \hat{\varphi}_G)| > 10'\}/N_G. \tag{5}$$

The concrete values $\delta_G^{\min}$ and $p_G^{\min}$ for different constellations G are listed below. The calculated values $\hat{\gamma}_G$ and $\hat{\varphi}_G$ are estimates of the real parameters γ_G and φ_G determining the group error. Though it is possible to prove some asymptotic properties of these estimates (see Theorem 1 below) we cannot consider $\hat{\gamma}_G$ and $\hat{\varphi}_G$ to be close to real values γ_G and φ_G because we do not have firm statistical reasons for such closeness, as the total number of stars in constellations does not exceed 20–30. Consequently, the values $\hat{\gamma}_G$ and $\hat{\varphi}_G$ can only serve to calculate a lower bound $\delta_G^{\min}$ for the mean-square latitude error in the constellation G. The value $p_G^{\min}$ gives us some additional useful information about group errors. We need a considerably larger group of stars to reliably estimate group error. It turns out that there are seven regions in the Almagest star atlas which differ one from another from the point of view of the measurement accuracy of latitudes. Each of these seven regions is 'homogeneous', i.e. the measurement accuracy in this region is more or less the same for most of the stars. This fact is very important. It was discovered in our computer experiments with the data from the Almagest star catalogue. We would like to note that the same division of the star atlas follows from systematization of the results of preceding researchers but that is also beyond the scope of this paper. here is a list of the seven regions (see Figure 3):

Region A contains all the stars ($N_A = 249$) of the northern part of the sky and of the zodiac which are located on the side of the Milky Way containing the spring equinox point.

Region B is a similar region ($N_B = 262$) located on the other side of the Milky Way.

Region ZodA contains all the zodiacal stars ($N_{ZodA} = 124$) from region A and consists of six constellations: Gemini, Cancer, Leo, Virgo, Libra, Scorpius.

Region ZodB contains all the zodiacal stars ($N_{ZodB} = 168$) from region B.

Region C contains all the southern stars ($N_C = 116$) located on the same side of the Milky Way as region A.

Region D contains all the southern stars ($N_D = 143$) located on the same side of the Milky Way as region B.

Region M is the Milky Way ($N_M = 94$).

More details are to be found in Table II.

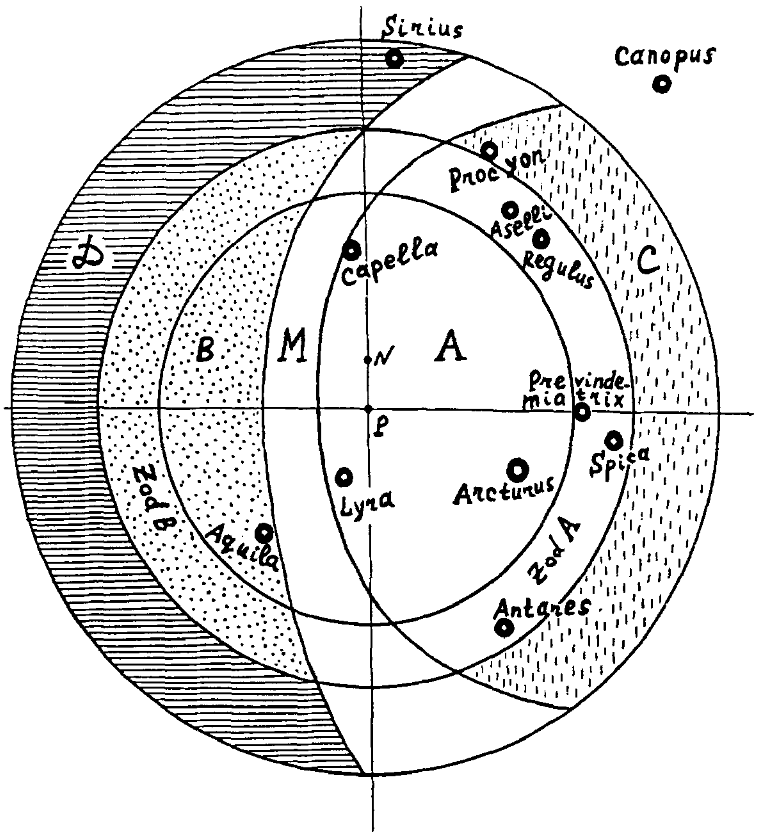

Fig. 3. The seven regions given in the Almagest star atlas.

Let us consider a 'large' group of stars R and determine the parameters $\hat{\gamma}_R$ and $\hat{\varphi}_R$ using the above relation (4) where one should replace G by R.

THEOREM 1. *Let us suppose that for all stars $i \in R$ parameters γ_i and φ_i are equal for all i (see* (1), (2)) *and coincide with γ_R and φ_R, respectively. Then the values $\hat{\gamma}_R$ and $\hat{\varphi}_R$ have the following properties*:

Table II.

Region (G)	Baily's number of stars from a region before cleaning up the catalogue	Total number of stars in a region after cleaning up the catalogue
A	1–158, 424–569	249
B	286–423, 570–711	262
C	847–997	116
D	712–846, 998–1028	143
M	159–285	94
ZodA	424–569	124
ZodB	362–423, 570–711	168

(1) *$\hat{\gamma}_R$ is a nonbiased estimate of the value γ_R having a normal distribution with a variation*

$$\delta^2(\hat{\gamma}_R) = d[N_R(s_{20}\cos^2\varphi_R + 2d_0\cos\varphi_R\sin\varphi_R + c_{20}\sin^2\varphi_R)]^{-1},$$

where

$$s_{20} = \left[\sum_{i=1}^{N_R}\sin^2 L_i(t)\right]\Big/ N_R,$$

$$c_{20} = \left[\sum_{i=1}^{N_R}\cos^2 L_i(t)\right]\Big/ N_R,$$

$$d_0 = \left[\sum_{i=1}^{N_R}\sin L_i(t)\cos L(t)\right]\Big/ N_R.$$

(2) The estimate $\hat{\varphi}_R$ is asymptotically (when $N_R \to \infty$) unbiased for the value φ_R, its distribution function can be calculated in terms of a normal distribution (we do not need a concrete formula here).

(3) The value $\delta_R^{\min}(t^*)$ is an asymptotically nonbiased estimate for the real mean-square error

$$d^{1/2} = (E\xi_i^2)^{1/2}$$

of measurements.

We shall call the parameters φ_R and γ_R systematic errors in the group R. The values $\delta_R^{\min}$ characterizes the exactness of measurements in the region R. Thus, in order to discover groups of well-measured stars, we can use the following algorithm.

Algorithm of the Choice of Well-Measured Groups of Stars

(1) Calculate the values $\hat{\gamma}_R$, $\hat{\varphi}_R$, $\delta_R^{\min}$ for each 'large' group R of stars;

(2) choose the group $R_0 = \operatorname{argmin} \delta_R^{\min}$;

(3) test that the calculated values $\hat{\gamma}_{R_0}$ and $\hat{\varphi}_{R_0}$ are really parameters of the group error for all individual constellations from R_0. Consequently, Theorem 1 is valid. All such constellations G form a set of well-measured stars. Of course, this set can be empty;

(4) delete the set R_0 from the initial catalogue and repeat the algorithm beginning with step 1, etc.

As a result we obtain the hierarchy of well-measured collections of stars corresponding to the accuracy of the latitude measurements.

Step 3 in the above algorithm will be discussed more fully in some comments which we shall give below.

Let us note that the epoch t of real observations is unknown to us. Hence, all conclusions made above have a conventional character (given that the catalogue was compiled in the epoch t). Consequently, we need to test all values t from our a-priori time interval. As we know, the trajectory of the real ecliptic pole from Newcomb's theory, it is sufficient to obtain $\hat{\gamma}_R$ and $\hat{\varphi}_R$ only for some fixed $t = t_0$. These two parameters determine the location of the 'catalogue ecliptic' and then give us the possibility of calculating $\hat{\gamma}_R$ and $\hat{\varphi}_R$ for all t.

6.4. ERROR VALUES IN THE ALMAGEST STAR CATALOGUE

Computer calculations resulted in the following values of minimal mean-square errors (they practically do not depend on t):

$$\delta_{\mathrm{A}}^{\min} = 16.5'; \qquad \delta_{\mathrm{B}}^{\min} = 19.2';$$
$$\delta_{\mathrm{ZodA}}^{\min} = 12.8'; \qquad \delta_{\mathrm{ZodB}}^{\min} = 19.3';$$
$$\delta_{\mathrm{C}}^{\min} = 22.5'; \qquad \delta_{\mathrm{D}}^{\min} = 24.4';$$
$$\delta_{\mathrm{M}}^{\min} = 20.5'.$$

It follows that the region ZodA is the most well-measured one in the star atlas. One can see the curve $\bar{\gamma}_{\mathrm{ZodA}}(t)$ (which, in fact is a line) in Figure 4. This curve is contained in the tolerance set corresponding to a confidence level $\epsilon = 0.05$. Similar curves were obtained for all the other regions. We also calculated all functions $\hat{\varphi}_R(t)$. An example can be seen in Figure 4. These calculations confirmed that the corresponding values $\hat{\beta}_R$ (which can be obtained from $\hat{\gamma}_R$ and $\hat{\varphi}_R$) are rather small ($|\hat{\beta}_R| < 5'$), i.e. $\beta \ll \gamma$.

But the tolerance sets for curves φ are very wide (about 40°). This fact indicates the 'nonsystematic' nature of the parameter φ. Indeed, the calculated value $\hat{\varphi}_{\mathrm{ZodA}}$ is only the average of individual values φ_G for six zodiacal constellations (from G_i = Gemini to G_6 = Scorpius). This fact can be considered as an indirect confirmation of the hypothesis that measurements were made by some instrument fixing an angle between the ecliptic and the equator (of course, with some error in the value of this angle). It is also probable that the axis of the rotation was fixed each time when a measurement occurred. One such ancient instrument is the well-known 'astrolabe' or 'armillary sphere' described by Ptolemy.

Now let us turn to the procedure of testing the hypothesis that the value $\hat{\gamma}_{\mathrm{ZodA}}$ determined by our calculations is common for all constellations from ZodA, i.e. this value really represents the group error. For each constellation G from ZodA, we calculate and compare the corresponding 'initial' error $\delta_G^{\min} = \delta_G(t, 0, 0)$, 'minimal' error $\delta_G^{\min}(t)$ and an error δ_G^{gr} which results after rotation over angles $\gamma_{\mathrm{ZodA}}(t)$ and $\varphi_{\mathrm{ZodA}}(t)$, i.e.

$$\delta_G^{\mathrm{gr}} = \delta_G(t, \gamma_{\mathrm{ZodA}}(t), \varphi_{\mathrm{ZodA}}(t)).$$

The result is shown in Figure 5 for t = AD 100. Similar calculations were made for all t. We can see from Figure 5 that the resulting effect induced by the 'optimal' individual rotation for each individual constellation, practically coincides with the effect induced by the 'common' rotation calculated for the total ZodA. We can

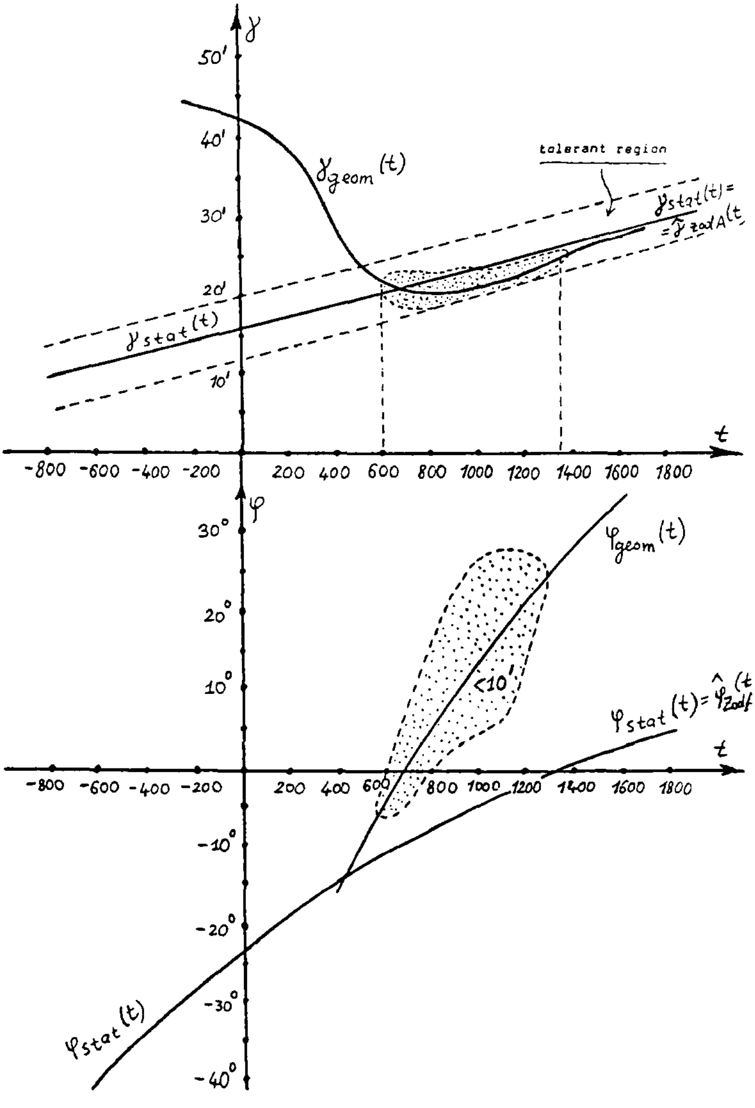

Fig. 4.

see also the additional confirmation of the group nature of the error $\gamma_{\text{ZodA}}(t)$ in Figure 6 where we demonstrate graphs of the percentages of the stars with latitude deviations not exceeding 10′ after corresponding 'optimal' rotation and without rotation (initial percentage).

We also investigated neighbourhoods of eight named stars: Antares, Arcturus, Aselli, Lyra, Capella, Procyon, Regulus, Spica. Two of these stars (Arcturus and Procyon), have a large velocity of proper motion.

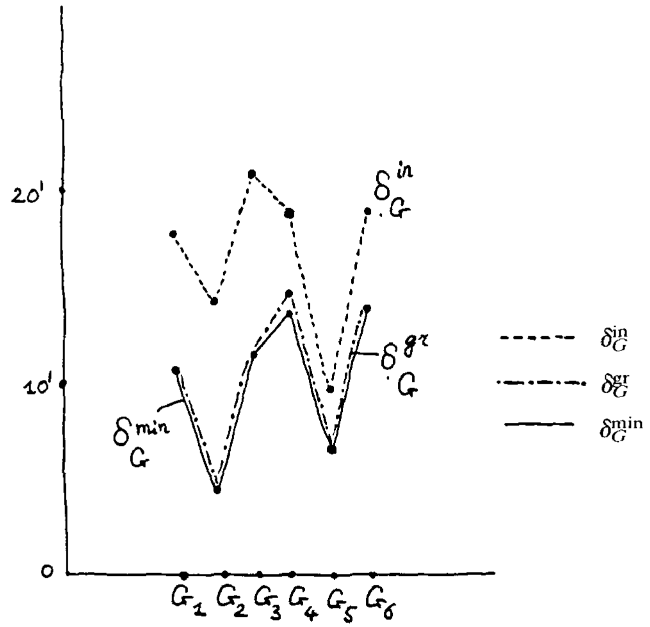

Fig. 5.

It turned out that the group errors for all these stars are the same (or very close) as the stars from ZodA. Numerical data contained in the star catalogue are not sufficient to determine reliable group errors for neighbourhoods of only two stars: Aquila and Sirius. This was the reason why we excluded them from further consideration.

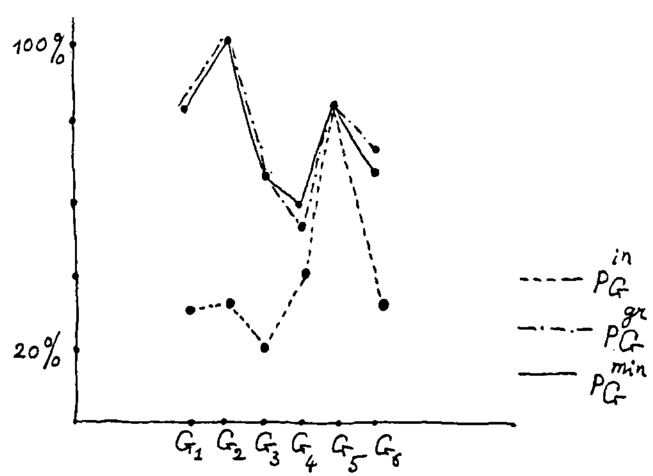

Fig. 6.

7. The Dating of the Almagest Star Catalogue

7.1. STATISTICAL DATING PROCEDURE

Let I be the set of eight named stars (see above) and

$$\Delta(t, \gamma, \varphi) = \max_{i \in I} |\Delta b_i(t, \gamma, \varphi)|.$$

We base our dating procedure on the hypothesis that the latitudes of *all* named stars from I must have *individual* errors of not more than 10′ in the year, t^*, of observations. In other words,

$$\Delta(t^*, \gamma, \varphi) \leq 10'$$

and the value γ belongs to the statistical tolerance interval (see Figure 4). Corresponding adoptable values of γ are 'marked' with points in Figure 4. Consequently, *we claim that the time-interval AD* $600 \leq t^* \leq$ *AD* 1300 *can be considered as a dating interval*. Of course, this interval depends on different parameters in general: claiming accuracy (10′), confidence probability ϵ, and some others. The stability of the method will be analysed in Section 8 below.

7.2. GEOMETRICAL DATING PROCEDURE

Though we have determined a dating interval some doubts about it can appear due to the statistical nature of some assertions. In reality, we based our assertions on the fact that group errors for neighbourhoods of the eight named stars are the same. This fact was proved with the help of statistics. Hence, there is some positive probability (though it is very small) that this fact is wrong.

Let us again consider the value $\Delta(t, \gamma, \varphi)$ and find for every t quantities:

$$(\gamma_{geom}(t), \varphi_{geom}(t)) = \operatorname*{argmin}_{\gamma, \varphi} \Delta(t, \gamma, \varphi)$$

and

$$\Delta_{\min}(t) = \Delta(t, \gamma_{\text{geom}}(t), \varphi_{\text{geom}}(t)).$$

These quantities depend only on the position of the eight named stars whereas $\hat{\gamma}_{\text{ZodA}}(t)$ and $\hat{\varphi}_{\text{ZodA}}(t)$ *do not depend on them* (they depend on the positions of all stars from ZodA). It is clear that $\Delta_{\min}(t) \leq 10'$ if AD $600 \leq t \leq$ AD 1300. But it turned out that $\Delta_{\min}(t) \leq 10'$ *if and only if* AD $600 \leq t \leq$ AD 1300 (see Figure 7). Besides, $\gamma_{\text{geom}}(t) \approx \hat{\gamma}_{\text{ZodA}}(t)$ for these t (see Figure 4). Hence, *this confirms without any statistical arguments that the above interval is a dating one. There do not exist a* γ *and* φ *such that the inequality*

$$\Delta(t, \gamma, \varphi) \leq 10'$$

holds when $t <$ *AD* 600 *or* $t >$ *AD* 1300. We confirmed also that the systematic error calculated with the help of statistics (using the coordinates of all stars from

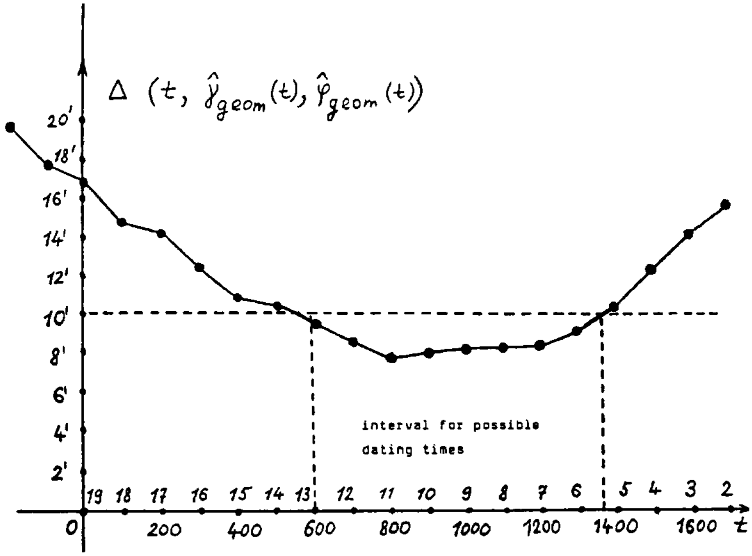

Fig. 7.

ZodA) is, in fact, 'geometrically optimal' for the eight named stars. Let us illustrate this result by means of Table III. The four stars 818, 288, 509, and 892 are the spikes which were previously removed from consideration.

Figure 8 shows graphs of individual latitude deviations dependent on t for the eight stars, given that $\gamma = 21'$, $\beta = 0$.

Table III.

	t					
No.	1	5	10	15	18	21
110	29.9	15.3	2.3	20.0	30.5	41.0
818	44.2	39.2	32.7	25.9	21.8	17.5
288	27.0	28.7	30.7	32.5	33.5	34.4
509	15.6	14.9	13.8	12.6	11.8	11.0
553	13.3	11.0	8.5	6.2	4.9	3.7
452	13.2	10.2	6.5	2.9	0.9	1.1
848	8.1	4.0	1.2	6.7	10.1	13.5
469	6.1	3.5	0.4	2.7	5.1	6.2
510	5.1	4.9	4.4	3.7	3.3	2.7
149	5.1	6.7	8.5	10.0	10.8	11.5
222	1.3	1.5	2.1	2.9	3.5	4.2
892	71.5	75.0	79.2	83.1	85.4	87.6

Hence,

(1) we confirmed the accuracy claimed by the compiler of the Almagest star catalogue;
(2) we calculated the time interval containing the actual date of observation. We also proved that the catalogue could not have been compiled (on the basis of actual observations) outside this time interval;
(3) we proved that the compiler made an error in the determination of the position of the ecliptic pole and calculated it ($\gamma = 20'$); besides he made an error in the determination of the position of the equator ($\beta < 5'$). It is also important to note that the systematic error γ explains the existence of a strange 'Peters' sinus' in latitude deviations for zodiacal stars [1, p. 6];
(4) we defined the information kernel (eight named stars) in accordance with the accuracy of the measurements of the coordinates.

8. Stability of the Method

8.1. Our calculations showed that the decreasing of the confidence probability ϵ (beginning from $\epsilon = 0.2$) does not shift the time interval of probable dating. We also obtained that this interval does not depend on the assumption about the normality of the distribution of random variables ξ_i (a kind of robustness).

8.2. Let us show how the final results depend on the content of the group of named stars (information kernel). Namely, let us consider a subset of this group. Of course, the dating time interval will be changed (more exactly, it will increase). For example, if we remove Arcturus (the fastest star in the group) then the left boundary of the dating time interval shifts to approximately AD 350 but it still does not touch traditional period of Ptolemy. Some useful information about the dependence on the content of the group of named stars is contained in Figure 9. There, for some fixed t, the empirical distribution functions

$$F^{(t)}(x) = \#\{i: |\Delta(i, t, \gamma, \varphi)| < x\}/12$$

are shown.

We see that the 'best' distribution function corresponds to $t = 10$ and $\gamma = 21'$. This confirms the assertion above.

8.3. Let us change the accuracy level Δ. Recall that we started with $\Delta = 10'$. Then the 'Ptolemy period' will be included only when $\Delta = 25'$.

8.4. Now consider not only the 'rigid rotation' of the celestial sphere (as group errors), but also an arbitrary diffeomorphism of the coodinates (which is, however, close to an identity mapping) reflecting possible distortions (deformations) of astronomical instruments. Then it occurs that we can reach the 'Ptolemy period' only for such deformations which are implied by about 4% deviations of real

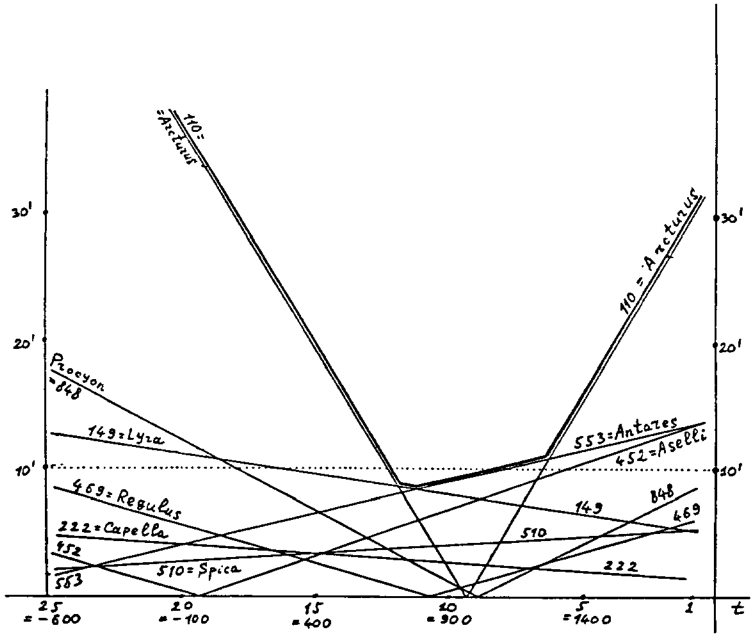

Fig. 8. Individual latitude deviations for named stars from the Almagest star catalogue.

instruments (e.g., the armillary sphere) from ideal ones. This is quite impossible, even for usual 'common' instruments (let alone scientific ones).

Consequently, our results are stable with respect to different deviations of both numerical data and the proposed assumptions.

9. Dating of Other Catalogues

9.1. TYCHO BRAHE'S CATALOGUE

The observations of T.Brahe were made at the end of the 16th century, but the catalogue was reduced to AD 1600.

Applying the above method to his catalogue, we discovered that Brahe did not make any significant systematic error in his determination of the position of the ecliptic pole ($\gamma \approx 1'$). This fact is not surprising for an astronomer working in the 16th century. On the other hand, the $1'$ accuracy claimed by Brahe is not achieved, even for named bright famous stars. In fact, his accuracy is about $2'$ for the set of his basic stars. For this catalogue, statistical and geometrical procedures lead to the same dating interval, from AD 1510 till AD 1620 containing the real observation epoch.

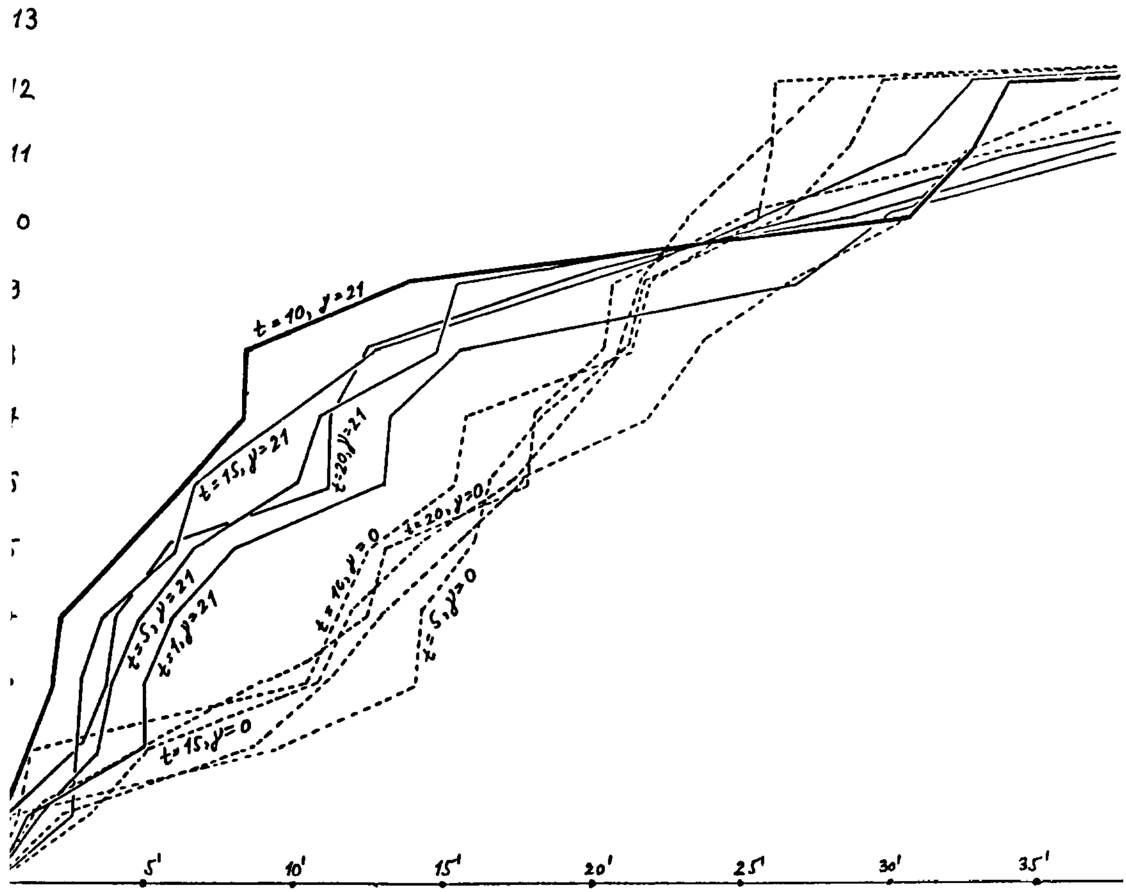

Fig. 9. Empirical distribution functions for the Almagest star catalogue. All 12 named stars are presented here. Continuous lines correspond to $\gamma = 21'$. Dotted lines correspond to $\gamma = 0'$. It is evident that $\gamma = 21'$ is much better than $\gamma = 0'$. The value $t = 10$ is optimal for $\gamma = 21'$ (even for $\gamma = 0'$).

9.2 HEVELIUS'S CATALOGUE

A similar dating interval was obtained for Hevelius's catalogue [5]. It was shown that the actual accuracy of latitudes in this catalogue is about 2′ but not 1″ as claimed by Hevelius. It is also possible that Hevelius not only used personal observations but also some other ones, perhaps those of Brahe.

9.3. ULUGBECK'S CATALOGUE

The application of the proposed statistical and geometrical procedures to Ulugbeck's catalogue [5] gave a dating interval AD 700–1400, which contains the real data of the compilation. Besides that, we found that a significant part of Ulugbeck's catalogue has a systematic error coinciding with that of the Almagest. It is very probable that this part was taken from the Almagest.

9.4. AL-SUFI'S CATALOGUE

As for al-Sufi's catalogue [22], it is, in fact, the Almagest star catalogue: latitudes for almost all stars are the same (even for the fast stars) and longitudes differ by 12°42′ exactly! So there is no need to date this catalogue.

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